Boundary controlled port Hamiltonian systems

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November 16th, 2015

Outline

1. Introduction - motivation

2. Short overview on boundary controlled port Hamiltonian systems

3. Considered acoustic system

4. Closed loop simulations

5. Conclusion and future work

Introduction - motivation

Some preliminary results on modeling and control of vibro-acoustic systems using the port Hamiltonian framework

Some preliminary results on modeling and control of vibro-acoustic systems using the port Hamiltonian framework

Port Hamiltonian systems

Class of non linear dynamic systems derived from an extension to open physical systems (1992) of Hamiltonian and Gradient systems. This class has been generalized (2001) to distributed parameter systems.

$$x(t): \begin{cases} \dot{x} = (J(x) - R(x)) \frac{\partial H(x)}{\partial x} + B(x)u \\ y = B(x)^T \frac{\partial H(x)}{\partial x} \end{cases} \qquad x(t,z): \begin{cases} \dot{x} = (\mathcal{J}(x) - \mathcal{R}(x)) \frac{\delta \mathcal{H}(x)}{\delta x} \\ \begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix} = \frac{\delta \mathcal{H}(x)}{\delta x}|_\partial \end{cases}$$

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- Central role of the energy.
- Additional information coming from the geometric structure.
- Multi-physic framework.

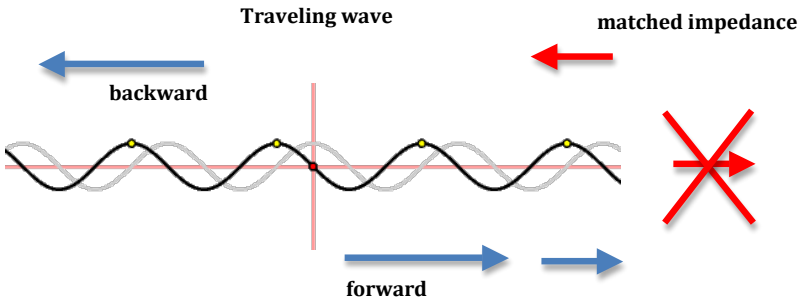
Introduction - motivation

Previous works

- Port Hamiltonian formulation of acoustic systems : 1D and 2 D wave equations [V.Talasila and der Schaft., 2002],
- Webster Lokshin equation (acoustic wave equation taking thermal and viscous effects into account) [Le Gorrec and Matignon, 2013]

$$\partial_t^2 w + \left(\varepsilon_z \, \partial_t^{1/2} + \eta_z \, \partial_t^{-1/2}\right) \partial_t w - \frac{1}{r_z^2} \partial_z \left(r_z^2 \partial_z w\right) = 0 \, .$$

- Matched impedance control vs. port Hamiltonian formulation



Aim

- First extension of boundary control to 2D linear systems
- Application to a very simple acoustic system

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1D wave equation

We first consider the boundary controlled 1D wave equation :

$$\begin{cases} \partial^2_{t^2}\theta - \frac{1}{\mu(z)}\partial_z\left(T(z)\partial_z\theta\right) = 0, \qquad z \in [0,1] \\ \theta(z,0) = \theta_0(z) \\ \partial_t\theta(z,0) = \theta_1(z) \\ \\ \partial_t\theta(0,t) = 0 \\ T(1)\partial_z\theta(1,t) = u_1(t) \end{cases}$$

with observation y_1 at the boundary $y_1(t) = \partial_t\theta(1,t)$. The total energy is given by :

$$E(t) := \frac{1}{2}\int_0^1 \left(\mu(z)\left(\partial_t\theta\right)^2 + T(z)\left(\partial_z\theta\right)^2\right) \mathrm{d}z\,,$$

and the energy balance by :

$$\frac{dE}{dt} = y_1(t)\,u_1(t)\,.$$

The system can be written $x = \left(\partial_z\theta \;;\; \mu\partial_t\theta\right)^T$

$$\frac{dx}{dt} = \begin{pmatrix} 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial z} & 0 \end{pmatrix} e = P_1\frac{\partial}{\partial z}\mathcal{L}x\,, \text{ with } P_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } e = \begin{pmatrix} T(z) & 0 \\ 0 & \frac{1}{\mu} \end{pmatrix} x$$

Considered class of PDEs (1D)

$$\frac{\partial x}{\partial t} = P_1 \frac{\partial}{\partial z}\left(\mathcal{L}(z)x\right)(t,z)) + (P_0 - G_0)\mathcal{L}(z)x(t,z)$$

$P_1 = P_1^\top$, $P_0 = -P_0^\top$, $G_0 \geq 0$, $x \in \mathbb{R}^n$, $z \in (a,b)$, $\mathcal{L}(z) = \mathcal{L}(z)^\top > 0$. State space $X = L_2(a,b;\mathbb{R}^n)$ with $\langle x_1, x_2\rangle_{\mathcal{L}} = \langle x_1, \mathcal{L}x_2\rangle$ and the norm $\|x_1\|^2_{\mathcal{L}} = \langle x_1, x_1\rangle_{\mathcal{L}}$.

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Boundary port variables

Let $\mathcal{L}x \in H^1(a,b;\mathbb{R}^n)$. Then the boundary port variables are the vectors $e_{\partial,\mathcal{L}x}, f_{\partial,\mathcal{L}x} \in \mathbb{R}^n$,

$$\begin{bmatrix} f_{\partial,\mathcal{L}x} \\ e_{\partial,\mathcal{L}x} \end{bmatrix} = U\frac{1}{\sqrt{2}}\begin{bmatrix} P_1 & -P_1 \\ I & I \end{bmatrix}\begin{bmatrix} (\mathcal{L}x)(b) \\ (\mathcal{L}x)(a) \end{bmatrix} = R\begin{bmatrix} (\mathcal{L}x)(b) \\ (\mathcal{L}x)(a) \end{bmatrix}.$$

Where

$$U^T\Sigma U = \Sigma, \quad \Sigma = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}, \qquad \Sigma \in M_{2n}(\mathbb{R})$$

Boundary controlled port Hamiltonian systems [Le Gorrec et al., 2005]

Let W be a $n \times 2n$ real matrix. If W has full rank and satisfies $W\Sigma W^\top \geq 0$, then the system $\frac{\partial x}{\partial t} = P_1 \frac{\partial}{\partial z}(\mathcal{L}(z)x)(t,z)) + (P_0 - G_0)\mathcal{L}(z)x(t,z)$with input

$$u(t) = W \begin{bmatrix} f_{\partial,\mathcal{L}x}(t) \\ e_{\partial,\mathcal{L}x}(t) \end{bmatrix}$$

is a BCS on X. The operator $\mathcal{A}x = P_1(\partial/\partial z)(\mathcal{L}x) + (P_0 - G_0)\mathcal{L}x$ with domain

$$D(\mathcal{A}) = \left\{ \mathcal{L}x \in H^1(a,b;\mathbb{R}^n) \;\middle|\; \begin{bmatrix} f_{\partial,\mathcal{L}x}(t) \\ e_{\partial,\mathcal{L}x}(t) \end{bmatrix} \in \ker W \right\}$$

generates a contraction semigroup on X.

Boundary controlled port Hamiltonian systems [Le Gorrec et al., 2005]

Let $\tilde{W}$ be a full rank matrix of size $n \times 2n$ with $\begin{bmatrix} W \\ \tilde{W} \end{bmatrix}$ invertible and let $P_{W,\tilde{W}}$ be given by

$$P_{W,\tilde{W}} = \left(\begin{bmatrix} W \\ \tilde{W} \end{bmatrix} \Sigma \begin{bmatrix} W \\ \tilde{W} \end{bmatrix}^\top \right)^{-1} = \begin{bmatrix} W\Sigma W^\top & W\Sigma \tilde{W}^\top \\ \tilde{W}\Sigma W^\top & \tilde{W}\Sigma \tilde{W}^\top \end{bmatrix}^{-1}.$$

Define the output of the system as the linear mapping $\mathcal{C} : \mathcal{L}^{-1} H^1(a, b; \mathbb{R}^n) \to \mathbb{R}^n$,

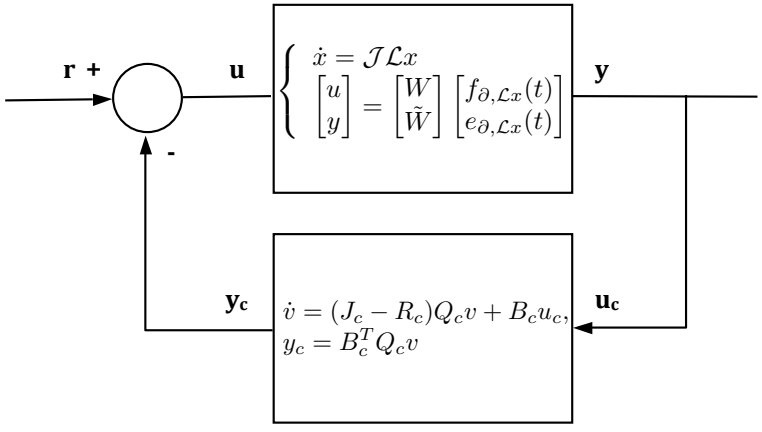
$$y = \mathcal{C}x(t) := \tilde{W} \begin{bmatrix} f_{\partial,\mathcal{L}x}(t) \\ e_{\partial,\mathcal{L}x}(t) \end{bmatrix}.$$

Then for $u \in C^2(0, \infty; \mathbb{R}^k)$, $\mathcal{L}x(0) \in H^1(a, b; \mathbb{R}^n)$, and $u(0) = W \begin{bmatrix} f_{\partial,\mathcal{L}x}(0) \\ e_{\partial,\mathcal{L}x}(0) \end{bmatrix}$ the following balance equation is satisfied :

$$\frac{1}{2}\frac{d}{dt}\|x(t)\|^2_{\mathcal{L}} = \frac{1}{2}\begin{bmatrix} u(t) \\ y(t) \end{bmatrix}^\top P_{W,\tilde{W}} \begin{bmatrix} u(t) \\ y(t) \end{bmatrix} - \langle G_0 \mathcal{L}x(t), \mathcal{L}x(t) \rangle \leq \frac{1}{2}\begin{bmatrix} u(t) \\ y(t) \end{bmatrix}^\top P_{W,\tilde{W}} \begin{bmatrix} u(t) \\ y(t) \end{bmatrix}.$$

Impedance passive case

As it has been pointed out in [Villegas, 2007], if the matrices W and $\tilde{W}$ are selected such that $P_{W,\tilde{W}} = \left[\begin{smallmatrix} 0 & I \\ I & 0 \end{smallmatrix}\right] = \Sigma$, then the BCS fulfills $\frac{1}{2}\frac{d}{dt}\|x(t)\|^2_{\mathcal{L}} \leq u^\top(t)y(t)$.



Assumption :

- $J_c = -J_c^T$, $R_c = R_c^T \geq 0$, $Q_c > 0$
- $(J_c - R_c)Q_c$ is Hurwitz

Asymptotic stability

- Existence of solution for the closed loop system + compactness property
- Lyapunov function + LaSalle's argument

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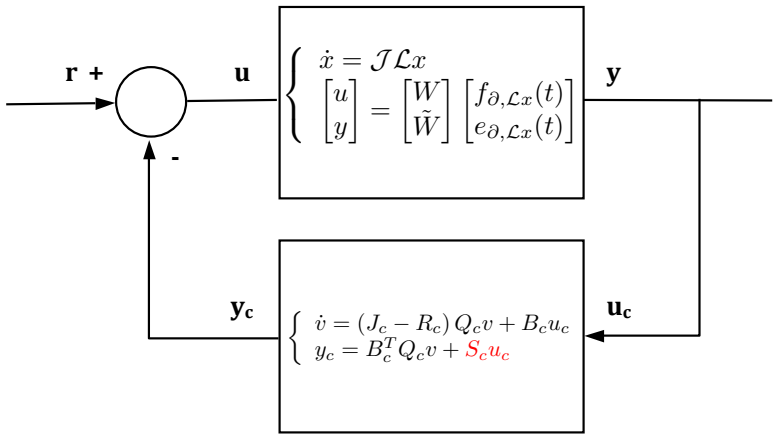

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- $(J_c - R_c)Q_c$ is Hurwitz
- Direct feedthrough $S_c > 0$

Exponential stability

- Existence of solution for the closed loop system + compactness property
- Lyapunov function

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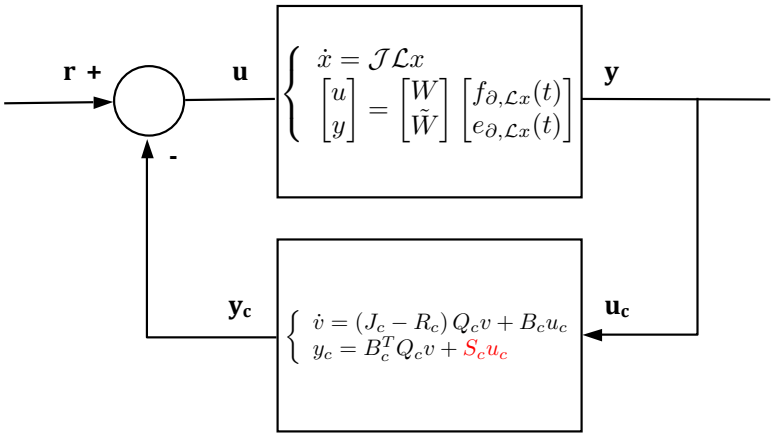

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Extensions to the non linear case

- Non linearity of the Hamiltonian function
- Existence, asymptotic and exponential stability

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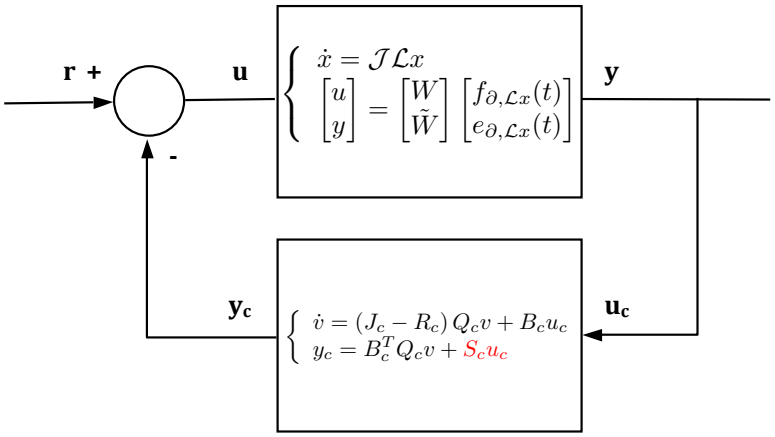

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Controller design

- Energy shaping through Casimir functions
- Imersion reduction scheme to get a boundary state feedback

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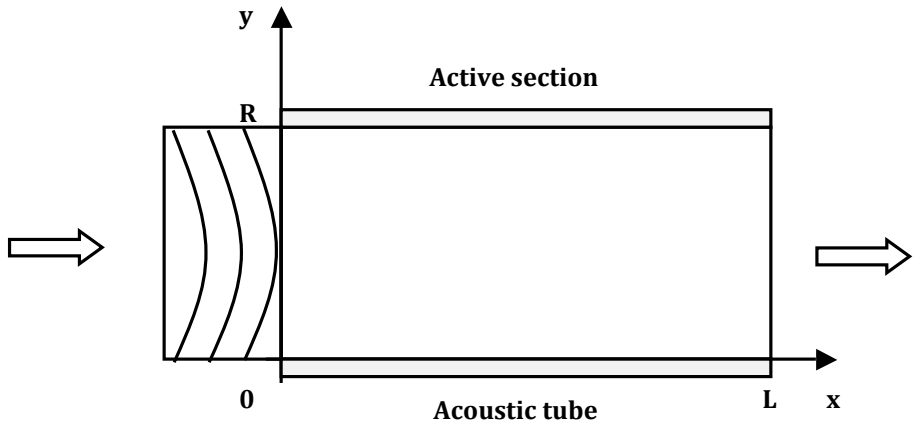
Considered acoustic system

We consider a partially actuated acoustic tube



$\mu(x,y,t) = \mu_0 + \mu_1(x,y,t)$: volume mass density
$v(x,y,t) = v_0 + v_1(x,y,t)$: velocity
$p(x,y,t) = p_0 + p_1(x,y,t)$: pressure

v_0, p_0, μ_0 : constant values.
v_1, p_1 , μ_1 : characterize the acoustic wave.

Considered vibro acoustic system

Assumptions

- No internal entropy creation (no internal energy dissipation).
- Acoustic approximation : v_1,p_1,μ_1 and their derivatives are infinitely small of the same order. No convection.
- Axysymmetric wave propagation$\longrightarrow v_y(x,0)=0$
- $\varsigma_{in}(0,y,t)$: input power density of the tube (at $x=0$), not controlled but bounded.
- Anechoic termination at $x=L$ boundary, (Collet et al.,2009) : $p_1(L,y)=c_0\mu_0 v_x(L,y)$ (c_0 : speed of sound in the air)

Remark

The 3D problem can be recast in a 2D problem using symmetries and changes of variables. In what follows we directly consider a 2D formulation.

Considered vibro acoustic system

Euler's equation / mass balance :

$$\mu_0 \dot{v}_1 = -grad(p_1)$$

$$\chi_s \dot{p}_1 = -div(v_1)$$

(χ_s : adiabatic compressibility factor)

Energy density :

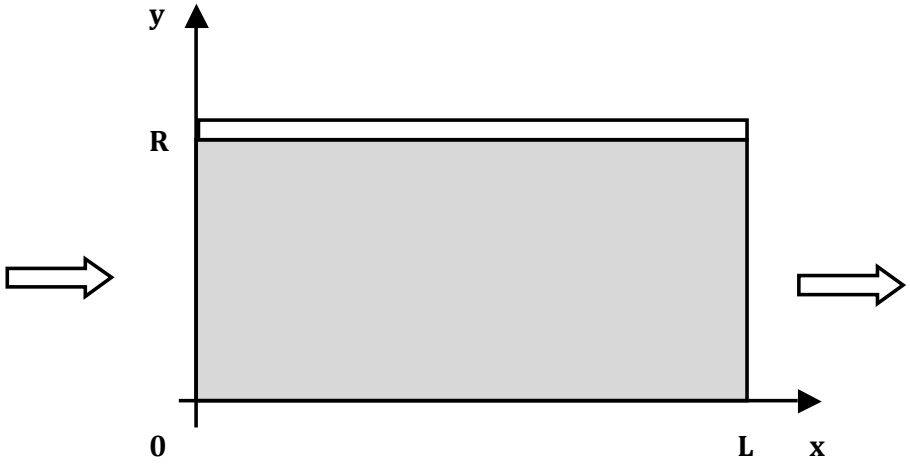
$$\varepsilon = \varepsilon_c + \varepsilon_p = \frac{\theta^\top \theta}{2\mu_0} + \frac{\Gamma^2}{2\chi_s}$$

The state variables are
$\theta = \mu_0 v_1$: density of quantity of movement,
$\Gamma = \chi_s p_1$: volume compression coefficient,
the energy variables of the system.

Using $z = \begin{pmatrix} \theta \\ \Gamma \end{pmatrix}$ we have :

$$\begin{pmatrix} \dot{\theta} \\ \dot{\Gamma} \end{pmatrix} = \begin{pmatrix} 0 & -grad \\ -div & 0 \end{pmatrix} \underbrace{\begin{pmatrix} \frac{\partial \varepsilon}{\partial \theta} \\ \frac{\partial \varepsilon}{\partial \Gamma} \end{pmatrix}}_{e = \begin{pmatrix} v_1 \\ p_1 \end{pmatrix}}$$

The 2D Wave Equation As A PHS



- Rectangular spatial domain $(x, y) \in [0, L] \times [0, R]$ with $<,>_{L_2^2}$ as $< v, w' >_{L_2^2} = \int_0^R \int_0^L v^T w' \, dxdy$
- 2*D* linear wave : $f = \mathcal{J} e$, $\mathcal{J} = \begin{pmatrix} 0 & -grad \\ -div & 0 \end{pmatrix}$
- $grad = \left(\frac{\partial}{\partial x} \ \frac{\partial}{\partial y} \right)^T$, $div = \frac{\partial}{\partial x} + \frac{\partial}{\partial y}$

Application To An Acoustic Process

$$f = \mathcal{J} e \Leftrightarrow \frac{d}{dt} \begin{pmatrix} \Theta_x \\ \Theta_y \\ \Gamma \end{pmatrix} = \begin{pmatrix} 0 & 0 & -\frac{\partial}{\partial x} \\ 0 & 0 & -\frac{\partial}{\partial y} \\ -\frac{\partial}{\partial x} & -\frac{\partial}{\partial y} & 0 \end{pmatrix} \begin{pmatrix} v_x(x,y) \\ v_y(x,y) \\ p(x,y) \end{pmatrix}$$

Port variables :

$$\begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix} = \begin{pmatrix} -e_3(L,y) \\ e_1(0,y) \\ -e_3(x,R) \\ e_2(x,0) \\ e_1(L,y) \\ e_3(0,y) \\ e_2(x,R) \\ e_3(x,0) \end{pmatrix} = \begin{pmatrix} -p_1(L,y) \\ v_{1x}(0,y) \\ -p_1(x,R) \\ v_{1y}(x,0) \\ v_{1x}(L,y) \\ p_1(0,y) \\ v_{1y}(x,R) \\ p_1(x,0) \end{pmatrix}$$

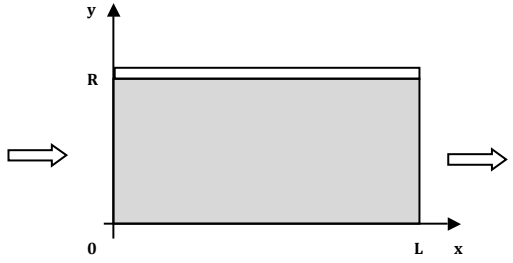
The total energy is :

$$\mathcal{H} = \int_0^R \int_0^L \left(\frac{\theta^\top \theta}{2\mu_0} + \frac{\Gamma^2}{2\chi_s} \right) dxdy$$

And the energy balance :

$$\dot{\mathcal{H}} = f_\partial^T e_\partial = \int_0^L (-e_3(x,R)e_2(x,R) + e_2(x,0)e_3(x,0))dx + \int_0^R (-e_3(L,y)e_1(L,y) + e_1(0,y)e_3(0,y))dy$$

Interconnection With The Active Surface



Infinite dimensions active surface :

$$\begin{cases} \dot{w}(x,t) = \begin{pmatrix} \frac{dq}{dt}(x,t) \\ \frac{d\varrho}{dt}(x,t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & -f(x) \end{pmatrix} \begin{pmatrix} k(x)q(x,t) \\ \frac{\varrho(x,t)}{m(x)} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u_1(x,t) \\ \bar{y}(x,t) = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} k(x)q(x,t) \\ \frac{\varrho(x,t)}{m(x)} \end{pmatrix} \end{cases}$$

Total energy : $\mathcal{H}_c(x,t) = \frac{1}{2} \int_0^L \left(k(x)q^2(x,t) + \frac{\varrho^2(x,t)}{m(x)} \right) dx$

Energy balance : $\dot{\mathcal{H}}_c = \int_0^L \left(u_1(x,t)\bar{y}(x,t) - f(x)\bar{y}^2(x,t) \right) dx$

Interconnection With The Active Surface

$e_2(x,R) = v_y(x,R) = \bar{y}(x,t)$: continuity of velocity
$e_3(x,R) = u_1 = -f_\partial(3)$: Newton's third law

Interconnected system :

$$\begin{cases} \begin{pmatrix} \dot{w} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} A & \mathcal{C} \\ 0 & \mathcal{J} \end{pmatrix} \begin{pmatrix} kq(x) \\ \frac{\varrho(x)}{m} \\ \frac{\partial \varepsilon}{\partial z} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} u_2(x,t) \\ e_2(x,R) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} kq(x) \\ \frac{\varrho(x)}{m} \\ \frac{\partial \varepsilon}{\partial z} \end{pmatrix} \end{cases}$$

$\mathcal{C}\left(\frac{\partial \varepsilon}{\partial z}\right) = \begin{pmatrix} 0 & 1 \end{pmatrix}^T e_3(x,R)$; $A = \begin{bmatrix} 0 & 1 \\ -1 & -f(x) \end{bmatrix}$

Taking the boundary conditions into consideration, the energy balance is :

$$\dot{\mathcal{H}}_T = \int_0^L (-f(x)e_2^2(x,R) + u_2 e_2(x,R))dx + \int_0^R (\varsigma_{in}(0,y) - \frac{1}{\mu_0 c_0} e_3^2(L,y))dy.$$

First control ideas :

- Passive control : damping injection

  $$u_2(x) = -k_2 e_2(x, R)$$

  Limited performances dues to the dynamics of the membrane.

- Emulation of the Anechoic condition (*i.e.* $e_3 = \mu_0 c_0 e_2$)

  $$u_2(x) = K(-\mu_0 c_0 e_2(x, R) + e_3(x, R))$$

  The performances depend on K.

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Closed loop simulations

- The system is supposed to be fully actuated at the upper boundary by an infinite number of loudspeakers represented by distributed mass-spring systems.
- The new input is the distributed force u_F applied to the loudspeaker membrane.
- The aforementioned control law is adapted to bypass the dynamics of the loudspeaker *i.e.* $u_F = K\left(-\mu_0 c_0 e_2(x, R) + e_3(x, R)\right)$

TABLE: The parameters of the tube [David et al., 2010]

L	Length	1.84 m
R	Radius	0.05 m
ρ	Air density	1.2754 kg/m^3
χ_s	Compressibility coefficient	1 Pa^{-1}

The actuation is applied along the second third of the tube

Closed loop simulations

We connect (in a power preserving way) the system with a dynamic controller of the form

$$\begin{cases} \dot{x}_c = (J_c - R_c)\frac{\partial H_c}{\partial x_c} + G_c u_c \\ y_c = G_c^T \frac{\partial H_c}{\partial x_c} + S_c u_c \end{cases}$$

Idea

The aim is to shape the closed loop energy function

$$H_{cl}(z, x_c) = H(z) + H_c(x_c)$$

through the use of structural invariant $C(z, x_c)$ in order to link x_c to z and a proper choice of $H_c(.,.)$

We look for Casimir functions of the form :

$$C(z, x_c) = \Phi^T x_c + \Psi(z)$$

such that

$$\frac{dC(z, x_c)}{dt} = 0$$

Energy shaping through Casimir functions

Assume that it is possible to find n_c independent Casimir functions,

$$\hat{\Phi} = (\Phi_1 \cdots \Phi_{n_c}) \quad \hat{\Psi} = (\Psi_1 \cdots \Psi_{n_c})^T$$

By assuming $\hat{\Phi} = -I$

$$x_{c,i}(t) = \Psi_i(z_1, z_2) + \kappa_i$$

And the total energy can be shaped by an appropriate choice of H_c such that :

$$H_{cl}(z, x_c) = H_{cl}(z) = H(z) + H_c(\hat{\Psi}(z) + \kappa_i), \quad \frac{dH_{cl}}{dz}(z^*) = 0$$

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Conclusion

- Geometric modeling of a 2D-wave equation system defined on a simple (rectangular) spatial domain
- First boundary control design
- 2D structure preserving discretization scheme issued from the primary works of [Golo et al., 2004].

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Ongoing and future work

- Characterization of the structural invariants suitable for control purpose
- Design of distributed control through finite network of actuators/sensors
- Real time implementation of the control

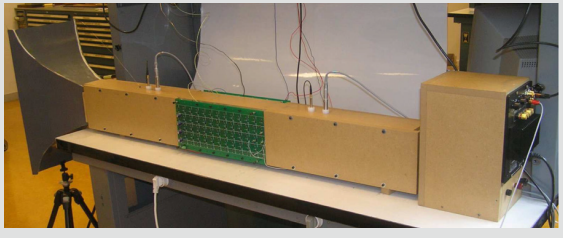

FIGURE: Experimental setup built at FEMTO-ST

Ongoing and future work

- Control at the boundary of more complex structures
- Toward boundary control of 2D non linear systems (when the convection is taken into account)

$$\begin{pmatrix} \rho \\ v \end{pmatrix} = \begin{pmatrix} 0 & -div \\ -grad & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{2}v^2 + \frac{\partial(\rho U(\rho))}{\partial \rho} \\ \rho v \end{pmatrix} \text{ with } U(\rho) = \frac{RT}{M} ln\,(\rho)$$

Ongoing and future work

- Control at the boundary of more complex structures
- Toward boundary control of 2D non linear systems (when the convection is taken into account)

$$\begin{pmatrix} \rho \\ v \end{pmatrix} = \begin{pmatrix} 0 & -div \\ -grad & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{2}v^2 + \frac{\partial(\rho U(\rho))}{\partial \rho} \\ \rho v \end{pmatrix} \text{ with } U(\rho) = \frac{RT}{M} ln\,(\rho)$$

Doctoral course
Introduction on Control of Port Hamiltonian Systems
FEMTO-ST - UBFC- ENSMM - UFC
March 23-25, 2016

Thank you for your attention !

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